

The Thirty-Seventh Annual Eastern Shore High School Mathematics Competition

April 27, 2022

Individual Contest Exam

Instructions

There are twenty problems on this exam. Select the best answer for each problem.

Your score will be the number of *correct* answers that you select.

There is no penalty for incorrect answers.

The use of a calculator is **not** permitted on this exam.

No computational work is required for any of your multiple choice responses.

However, in the event of tie scores, after the multiple choice responses have been checked for problems 1-20, the responses and/or written computational multiple choice responses for problems 21-40 will be checked.

1. Evaluate the following expression:

$$\log_4(\log_3(\log_2 8))$$

- a. 0 b. 1 c. 3 d. 4 e. 8

2. Suppose for two functions $f(x)$ and $g(x)$, $(f \circ g)(x) = |x| + 4$. The domain of $g(x)$ is the set of all real numbers. $f(x)$ and $g(x)$ could be which of the following?

- a. $f(x) = |x|$, $g(x) = x + 4$.
b. $f(x) = |x|$, $g(x) = x - 4$.
c. $f(x) = |x|$, $g(x) = -x + 4$.
d. a, b, and c

9. If $f(x) = (x - 3)(x - k)$ and if the remainder is 16 when $f(x)$ is divided by $(x + 1)$, then what is k ?

- a. -2 b. 0 c. 2 d. 3 e. 4

10. If $x^2 - 5x + 6 = 0$, what does $x^4 - 10x^3 + 26x^2 - 5x + 2$ equal?

- a. 2 b. 3 c. 24 d. 32 e. 36

11. Suppose a, b are nonzero real numbers and $a < b$. Which of the following statements are always true?

- a. $a^2 < b^2$ b. $a^3 < b^3$ c. $\frac{1}{a} > \frac{1}{b}$ d. Both b and c e. a, b, and c

12. What interval contains the solutions to the equation $\sqrt{2x+1} = 1 + \sqrt{x}$?

- a. [-2;3] b. [0;5] c. [2;7] d. [4;9] e. [6;11]

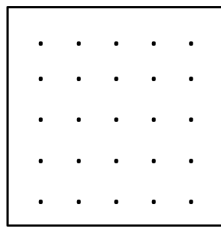
13. Find the root of the equation $2 + \log(\sqrt{1+x}) + 3 \log(\sqrt{1-x}) = \log(\sqrt{1-x^2})$.

- a. $\frac{9}{10}$ b. $\frac{99}{100}$ c. $\frac{1}{9}$ d. $\log \frac{9}{10}$ e. $\log(2)$

14. If $x > 0$, $x^2 = 2^{64}$ and $x^x = 2^y$, then what is the value of y ?

- a. 2 b. 2^{11} c. 2^{32} d. 2^{37} e. 2^{64}

15. Consider a 5 × 5 rectangular grid (as shown below) where the distance between any two consecutive horizontal or vertical lattice points on the grid is one unit. If two of these points are used as endpoints of a segment, segments of various lengths can be drawn. If only one segment of each possible length is drawn on the grid, what is the sum of the lengths of segments with integral lengths?



- a. 3 units b. 6 units c. 10 units d. 15 units e. 21 units

16. The respective coordinates of points $A; B; C;$ and D are $(0;0); (0;10); (100;15)$ and $(100;0)$. Segments AC and BD intersect at point E . Determine the area of triangle AED .

- a. 60 square units
b. 120 square units
c. 180 square units
d. 240 square units
e. 300 square units

17. A cat toy consists of 3 detachable balls: one catnip, one silvervine, and one gall fruit. The toy's description says that 70% of cats like catnip, 80% of cats like silvervine, and 90% of cats like gall fruit. What is the approximate chance that a cat will be interested in the toy (i.e., be attracted to it)?